WAVE INDUCED BENDING MOMENT DUE TO SHIP SLAMMING

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ABSTRACT

A mathematical formulation of the overall ship's response to bottom impact slamming in regular waves is developed. The hydrodynamic problem concerning the definition of the loads is first discussed, and a particular physical model is adopted for finding the hull vibratory behavior.

Based on the given formulation a general procedure leading to the time history representation of the midship's bending moment is suggested. Finally, an illustrative example of application to a Mariner ship is described and the results compared with available data.

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TABLE OF CONTENTS

NOMENCLATURE

A = ship's sectional area ft^2

B = ship's beam ft

c = damping coefficient per unit length ton-sec/ft^2

$\bar{c}_i$ = i^{th} mode generalized damping ton-sec/ft

c_s = structural damping per unit length ton-sec/ft

E = modulus of elasticity ton/ft^2

g = acceleration of gravity ft/sec^2

I_r = mass moment of inertia of hull per unit length with respect to an axis normal to the x-y plane; ton-sec^2

I = sectional area moment of inertia ft^4

KAG = hull's shear rigidity ton

$\bar{k}_i$ = i^{th} mode generalized spring constant ton/ft

L = ship's length ft

LBP = length between perpendiculars ft

m = added mass per unit length ton-sec^2/ft^2

m' = ship's mass per unit length ton-sec^2/ft^2

M = bending moment ft-ton

M_i = i^{th} mode moment spatial weighting function ton

P = total slamming load per unit length ton/ft

P_1 = inertial component of P per unit length ton/ft

P_2 = buoyancy component of P per unit length ton/ft

q_i = i^{th} mode generalized deflection ft

Q_i = i^{th} mode generalized forcing function ton

t = time variable sec

T = ship's draft ft

U = ship's speed ft/sec

V = shear force in y direction ton

V_i = i^{th} mode shear spatial weighting function ton/ft

v^* = threshold velocity ft/sec

w = regular wave frequency rad/sec

w_e = frequency of encounter rad/sec

w_i = i^{th} mode natural frequency rad/sec

WL = wave length ft

x = longitudinal position along the hull ft

X_i = i^{th} mode shape, dimensionless

y = vertical elastic deflection normal to x ft

z = relative motion between the hull and the wave surface ft

$\dot{z}$ = relative velocity between the hull and the wave surface ft/sec

μ = effective mass per unit length ton-sec^2/ft^2

$\bar{\mu}_i$ = i^{th} mode generalized mass ton-sec^2/ft

γ = component of the slope of y due to bending only rad

ρ = density of sea water lb-sec^2/ft^4

ϕ = phase angle of z and $\dot{z}$ motions rad

I. INTRODUCTION

Under certain severe sea conditions the phase relation between the bow motion and the surface of the oncoming waves is such that an impact may occur between the ship structure and the water. This impact of any portion of a moving ship upon the surface waves is commonly called "slamming". It is in most of the cases a result of large pitch and heave motions that force the ship's forward bottom to emerge and re-enter the water after hitting its surface. In general this is known as "bottom impact slamming". However, bottom impact is not a required condition for slamming to occur, and we can talk of bow flare slamming when there is a sudden change of the acceleration of the ship's bow without its actual emergence. Finally, as a third type of violent interaction between the ship's hull and the seaway we can consider waves breaking over the deck, sometimes called "shipping of green water".

In all cases the impact is such that certain portions of the ship's structure generally forward have to sustain heavy impulsive loads from the water, possibly causing serious local damage along with transient vibrations of the hull. The danger of heavy damage or high vibratory response is generally reduced at sea by changing course or speed, or both, and this means that slamming not only poses a serious threat to safe navigation but also results in a considerable loss of time that affects the operational capabilities of a ship.

At the design stage slamming should be studied so that its effects on the ship structure and operation can be conveniently estimated. This study could eventually result in an improved design, with better handling characteristics, better structural reliability and more precise rules for what to do at sea in order to find an acceptable equilibrium between safety and effectiveness.

Essentially we can consider two problems when dealing with the ship's response to slamming, a localized one, also called a "micro", and an overall or "macro-problem". The localized approach deals mainly with local plate forces and damage resulting from direct application of the load and it tends to be a fairly sophisticated hydro-aerodynamic problem with plastic structural analysis playing a possibly important role [1].* The overall response involves hull vibrations and large midship stresses and bending moment that can be detrimental to the structure as a whole. Following the initial localized response and before an overall effect is felt we can also define a transition period in which the stress wave is travelling along the hull. This logical breakdown of the slamming response is useful since it not only makes a study approach easier but also tends to establish a clearer boundary between the two main sciences involved, hydrodynamics and structural mechanics.

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*Numbers in brackets [] designate references.

This study will focus on the overall ship response to bottom impact slamming, which is the type more likely to produce a real threat to the ship's structure. The reason for treating the problem from the macro point of view is that most of the investigations on bottom impact have so far dealt with local effects [2], but as ship's dimensions and speed requirements increase the overall response gains a stronger importance.

In our attempt to make a brief review of what we consider the most significant papers written on the particular field of overall response to bottom impact, we point out first the paper by Andrews [3] where a method for determining the elastic body response of a ship to a seaway was developed. The ship was divided into equal sections and for each one a forcing function and mass-elastic parameters representing the hull were computed. The forcing function was considered to include an unsteady hydrodynamic component obtained from the measured rigid body motions, and a hydrostatic component. The ship was essentially treated as a free-free beam and the waves were assumed to be trochoidal. The damping coefficient was given the value zero throughout the computations.

The work of Leibowitz [4, 5] represents a refined extension of Andrews' report with more precise treatments for certain parameters, like added mass, description of the waves and damping. The computations were based on an experimental

knowledge of the ship's rigid-body motions relative to the waves, which were taken from graphical records of periods of time where slamming was actually known to have occured.

The need to represent a ship by a more realistic model other than the pure free-free or Timoshenko beam has been extensively studied from the vibrations point of view by several authors who will be referred to later. Kline and Clough [6] specifically attempted the use of an improved model to study the dynamic response of the hull, including slamming. The influence of bulkhead spacing, machinery and cargo location, hull girder and double-bottom stiffness were particularly analyzed by these authors. Their model included a main elastic axis representing the primary flexural behavior of the hull girder and an additional axis accounting for the double bottom, which is in fact capable of independent local deformations. At intervals the two axes were coupled either by rigid connections representing the bulkheads or by springs allowing a certain degree of relative movement of the two girders. Finally, buoyancy springs provided a support to the model. The slamming load or forcing function was assumed to be a half sine wave impulse applied to the flexible bottom structure at a particular station near the bow.

The work of Kaplan [7] evaluates existing mathematical models describing ship-wave interaction, and it is mostly concerned with the computer simulation of wave-induced

structural loadings, including not only bottom impact slamming but also bow flare slamming and springing, or wave-excited main hull vibration. The structural model adopted is the free-free beam and a definition of the bottom impact forcing function similar to the one given by Andrews and Leibowitz is followed. The author concludes it is not in practice convenient to represent the bending moment due to slamming in a spectral form, and suggests as more effective and correct the time history representation, which can be directly combined with the time history output due to wave-induced effects. The computational method requires a hybrid linkage system that complements the digital computer. The random wave input is obtained through a white noise generator adequately filtered to produce a desired spectral form.

A probabilistic approach to bottom impact slamming has been made by Ochi [8, 9, 10] but its analysis does not exactly fit the "overall" concept, in the sense used here of hull vibrations and large midship bending moment. Ochi essentially finds expressions for the probability density functions of occurrence of slamming, of impact pressure applied to the ship's forward bottom, and of extreme slamming pressure. These results are mainly of interest when studying local bottom plate damage, and the overall study as understood here should, on the other hand, provide us ultimately with the characteristics of the midship's bending moment as a random process.

Besides the problem of the overall ship response to bottom impact slamming, for which we just presented the most representative existing investigations, reference should also be made to springing or wave-induced main hull vibration, since these two subjects show strong analogies. In fact, both seek an overall structural response that involves hull vibrations resulting from ship-waves interaction, and both require a suitable physical modeling of the ship.

Goodman [11] attempted to find a mathematical method capable of solving the springing problem and got results for regular waves and irregular seas. The beam model was used based on the assumption that the two mode model of vibration predominates with a small concentration of wave energy in the higher modes.

Van Gunsteren [12] develops also a theory for springing using a beam model in which the mass and stiffness distributions are discretized. His approach is similar to the one of Goodman but the numerical results do not completely agree.

From the considerations of the existing studies on ship overall response to bottom slamming we may make some general conclusions. First, a general method capable of evaluating the response characteristics of a ship in the design stage does not effectively exist. Second, the possible theoretical models may become extremely complicated if all important parameters are to be included and so simplifications are necessary. Third,

two main technical areas are involved in the problem, specifically hydrodynamics and structural mechanics, reflecting somehow the complexity already pointed out.

The purpose of this study is the development of a simple and convenient mathematical model capable of effectively defining the ship's overall response to bottom impact slamming. The desired mathematical model is such that not only new designs can be tested as far as slamming performance is concerned, but also a best design may eventually be achieved.

The hydrodynamic problem of slamming will first be treated, followed by the structural problem of ship hull vibration. Finally a numerical example of application will illustrate the proposed method.

II. BOTTOM IMPACT SLAMMING

1. NECESSARY CONDITIONS

Several authors have studied and tried to formulate mathematically the conditions that are required for bottom impact slamming to occur. The leading results we will refer to were given by Tick and Ochi.

Tick [13] established three necessary conditions:

a) The relative velocity between the bow and the sea surface must exceed a critical amount at the time of contact.

b) The bow must be out of the water previous to the contact.

c) The angle between the wave at some chosen contact point and the keel line must be small.

From tests conducted in regular waves Ochi [14] inferred the three following requirements:

a) bow (forefoot emergence);

b) certain magnitude of vertical relative velocity between ship bow and wave;

c) unfavorable phase between bow and wave motions.

After conducting tests in irregular waves Ochi could reduce the number of his previous necessary conditions by one, obtaining the following two:

a) bow (forefoot emergence);

b) a certain magnitude of relative velocity between wave and ship bow.

Ochi claims that his first condition is in fact a prerequisite for slamming since tests revealed that slamming never occured without bow emergence. This was found to be valid no matter what sea state, ship course and speed or loading distributions. Being a necessary condition, it is not, however, a sufficient one, and a certain magnitude of relative velocity between ship bow and wave was found to also be required. Ochi called this critical relative velocity below which slamming does not occur the threshold velocity, which we denote by v* (ft/sec). Tests conducted by this author under several conditions with a MARINER model showed that the value of the threshold velocity is nearly constant with an average of 12 ft/sec for a ship 520 ft long. This value should be modified according to the Froude scaling law for ships of different lengths.

Ferdinande [15] further investigated the nature of the threshold velocity and found in his tests that it was dependent on ship speed for different levels of slamming severity. The lower the ship speed the higher the threshold velocity for the same severity of slamming. This author also pointed out that this threshold velocity as defined by the Ochi probabilistic analysis was not really a strict minimum value

of re-entry velocity under which no slamming of the considered degree of severity could occur, and he considered it a rather artificial concept. Finally he also showed that to some extent the squared threshold velocities increased in a linear way with increasing slamming severity, related in this case to deceleration.

On his investigations on the circumstances leading to slamming, Ferdinande found that a slam could occur after a partial emergence of the keel, but with the forefoot being already immersed. Also a few slams of a lower degree of severity occurred at a moment the whole keel and the forefoot seemed to be immersed. The final contact of the bottom with the wave surface could be located at an appreciable distance from the forward perpendicular, but for some slams the exact location of impact was difficult to define.

Another author, Aertssen [16], considers the value of the threshold velocity as given by Ochi to be very low, and suggests that it should be taken 50%, or 18 ft/sec, greater for the MARINER.

We may conclude that there is still a certain degree of controversy on the exact definition of the conditions leading to slamming. Ochi [9] also admits that his threshold velocity, which was of course found empirically through experiments, has not yet been given a rigorous explanation. Anyway his two requirements have so far given satisfactory results,

and these show a good agreement with practical observations related to the probabilistic approach already described. For this reason and because they are simple to observe and control we will adopt them in our analysis.

We conclude this section giving a more precise definition of the conditions leading to slamming.

By $z(x,t)$ we denote the relative motion and by $\dot{z}(x,t)$ the relative velocity between the hull and the wave surface, both functions of x, a coordinate defining the position along the hull, and t, the time instant.

The coordinate system used throughout this work to define the geometry of the ship is fixed at the calm water level with the origin at midships. The x axis coincides with the ship's longitudinal axis and its positive direction points to the bow. If L is the length of the ship then $-L/2 \leq x \leq L/2$. The positive z axis points down (this vertical axis will be called later y when considering the vibration problem).

Corresponding to the bow emergence and threshold velocity conditions we can write

$$\begin{aligned} z(x,t) &\geq T \\ |\dot{z}(x,t)| &\geq v^* \end{aligned} \tag{1a}$$

T is the ship's draft at the section where we want to investigate if the necessary conditions are or are not satisfied.

Since bottom impact slamming involves bow emergence, we can evaluate (1a) at a selected forward location, like the

forward perpendicular or station 1, when trying to conclude whether the ship is experiencing slams or not. Anyway, attention should also be given to other locations along the hull towards midships, where conditions (1a) can both be met while not being so evident at the forward perpendicular.

Finally we adopt for the threshold velocity the figure suggested by Ochi, $v^* = 12$ ft/sec for a ship 520 ft long with an appropriate Froude scaling law for ships of different dimensions, or

$$v^* = 0.53\sqrt{L} \tag{1b}$$

2. THE SLAMMING LOAD

The exact description of the slamming forcing function is perhaps the most difficult problem of all this analysis and it has not yet been completely explained. Here the distinction between localized and overall effects tends to be more difficult and a greater degree of simplification and assumptions must be followed. Again, most of the investigation that has been done has mainly focussed on the local impact problem, generally through experimental techniques and empirical methods, rather than rigorous ones.

The first slamming theories [16] were of the wedge impact type and were of particular interest for the study of the landing of seaplanes. The greatest criticism that can be made is that they simply neglected the effects of compressibility

of both air and water, which tends to become a rather gross approximation.

More recently [17] it has been shown that the air layer between the falling body and the surface of the water plays a dominant role in the impact phenomenon. The air escape from under the body, depending on its shape, is not complete and a large bubble provides some kind of cushioning to the impact. Under the middle of a flat bottom model the water surface is forced to go down by the action of the air, while at the same time it rises at the edges, so that the contact is actually made before the body reaches the undisturbed surface. After the impact the situation is more difficult to analyze because further air bubbles are produced and more realistic three-dimensional tests are hard to control.

Chuang [18] showed that the cushioning effect of the compressible air trapped between the descending body and the water surface reduces the maximum impact pressure to about one-tenth of the acoustic pressure, and so its presence cannot really be ignored when local damage is considered.

In a more recent paper Chuang [19] reports the results of a series of drop tests with cone-shaped models intended to study the three-dimensional effects of slamming.

The existing studies are somehow difficult to extend to the real problem of ship bottom impact, because no account is taken of forward speed or of the three-dimensional flow

caused by changes in the shape of the sections. Besides, other important factors have been either disregarded or not considered in full detail, like shape, velocity and phasing of the surface waves, geometry of the hull and hull section characteristics, air and water compressibility, two- and three-dimensional air and water flow, bubbles and spray.

We do not know yet for sure which mechanisms are capable of playing a dominant role in the impact process and for this reason a high degree of simplification must be introduced. The factors so far considered tend to be in general important from the local point of view and the overall response may be simply based on a fairly straightforward physical concept. This may be well accepted if the impact process is further broken down, as proposed by Oakley [20], into three main phases. In phase I the body is approaching the free surface until the moment the first contact is made. During this period of time the air flow and the wave surface deflection are of predominant importance. In phase II the body impacts fully on the surface and penetrates it until a more or less wetting is achieved. The cushioning effect and the air spray may be the prime factors, as well as the water flow around the body. Finally in phase III we have the fully wetted problem where we can consider the pressures to be static and the forces to be the result of the rate of change of momentum.

It is difficult to draw the border between the above phases, mainly phases II and III, but we can admit that the

ship response is a result of high loads generated as the ship re-enters the wave surface and displaces a large amount of water. The physical definition of the forcing function becomes rather obvious if we use the concept of added mass and state that part of this force is due to the rate of change of the momentum of the added mass of the hull which dives into the wave [21]. We will call this the inertial or hydrodynamic effect of the slamming load.

As the ship penetrates the water another force is generated because of the increasing buoyancy of the system, and we call this the buoyancy or hydrostatic effect component of the slamming load. A more realistic representation of the buoyancy term would include the so-called Smith effect, which accounts for the centrifugal forces set up by the orbital motion of the particles in the wave, which make the wave pressure not conform exactly to the laws of hydrodynamics.

This model transforms a rather complex problem into a fairly simple form which may be incomplete but has the advantage of being easily usable in specific numerical applications, which is our final aim. Local effects like air cushioning and spray and lateral components of the loads are neglected to give a time varying vertical force that has been shown to provide acceptable results by Leibowitz [4] and Kaplan [7].

Reference should again be made to another possible treatment of this problem as made by Kline and Clough [6], which

tends to be rather oversimplified. In fact, these authors simply represent the slamming load by a half sine wave impulse applied to the bottom structure at a specified forward station. Three durations of loading are considered for the same total impulse.

To conclude the discussion on the bottom impact slamming load we will formulate it in mathematical terms so that it can be conveniently used to obtain the overall response of the ship.

3. MATHEMATICAL FORMULATION

As already stated, the total slamming load, which we denote by $P(x,t)$ is the sum of an inertial term $P_1(x,t)$ and a buoyancy or hydrostatic term $P_2(x,t)$. These forces are defined on a per-unit length basis and they are a function of both time and location along the ship's longitudinal axis.

As defined before, $P_1(x,t)$ is the rate of change of the momentum of the hull's added mass $m(x,t)$. In mathematical terms this can be written as

$$\begin{aligned} P_1(x,t) &= \frac{D}{Dt}(m\dot{z}) \\ &= \frac{\partial}{\partial t}(m\dot{z}) + \frac{\partial}{\partial x}(m\dot{z})\frac{dx}{dt} \qquad (2) \end{aligned}$$

In the above definition the total derivative D/Dt is used since the rate of change of momentum occurs not only with respect to time but also with respect to x. The added mass

per unit length $m(x,t)$ is calculated for a certain frequency of encounter w_e, and it is essentially a function of the shape of the immersed part of the section, which is itself a function of x and t.

The frequency of encounter w_e (rad/sec) is defined in terms of the wave frequence w (rad/sec), the ship's speed U (ft/sec) and the acceleration of gravity g (ft/sec^2) or

$$w_e = w + \frac{w^2}{g} U \qquad (3)$$

The term dx/dt in (2) is the relative horizontal velocity of the water with respect to the ship. If we neglect the particle horizontal velocity, which is in general small, then $dx/dt \approx -U$, and $P_1(x,t)$ becomes

$$P_1(x,t) = \frac{\partial}{\partial t}(m\dot{z}) - U\frac{\partial}{\partial x}(m\dot{z}) \qquad (4)$$

The buoyancy contribution to the total force per unit length, not considering the Smith effect correction, is given by

$$P_2(x,t) = \rho g A \qquad (5)$$

ρ is the density of sea water and A is the immersed cross sectional area of the ship at a particular location x.

Combining equations (4) and (5) we obtain the final form of the total slamming load per unit length for a particular location x:

$$P(x,t) = \frac{\partial}{\partial t}(m\dot{z}) - U\frac{\partial}{\partial x}(m\dot{z}) + \rho g A \qquad (6)$$

Note that in this expression both A and m are functions of x and t, since the wave surface position is continuously changing relative to the hull.

III. SHIP HULL VIBRATION

1. PHYSICAL MODEL

The problem of ship hull vibration is essentially one of finding a physical model that conveniently represents the hull's structural behavior and is capable of giving accurate or acceptable information about the vibratory response of the ship.

Most of the work on ship hull vibration has treated the ship as a free-free beam with the equation governing the motion being the Timoshenko equation which incorporates the bending, shear and rotary inertia effects. Several numerical methods have been used to solve this equation or system of equations in order to calculate the natural frequencies and mode shapes, and the results agree well with measurements taken aboard ships except for the higher modes above the four-node mode, where some discrepancies arise. In general, the calculated values of the natural frequencies increase faster than the measured ones. This fact has been explained by several arguments, mainly in [22]:

a) inaccurate evaluations of structural parameters like stiffness, load distribution, added mass, etc.;

b) vibration has been affected by other factors which cannot be accounted for by the simple beam theory;

c) the assumption that the ship vibrates like a beam no longer holds after a certain mode.

Hylarides [23] points out shear lag as an important factor not taken into consideration by the traditional beam approach.

Also, the beam theory simply ignores the coupling between the horizontal and torsional motions and it does not consider the influence of local vibrating parts of the hull. Finally, we may say that some of the shortcomings of the beam method are caused by neglecting the three-dimensional character of the ship's structure, like structural discontinuities and three-dimensional mass distribution, substituting for it a somehow oversimplified physical model.

Several attempts have been made in order to improve the beam modeling of ships for vibrational response studies. Among others, we may refer to the three-dimensional beam-shell-sprung body model of the NS SAVANNAH, with the propeller and shafting treated as a sprung body attached to a beam-shell system representing the hull [24].

Kline and Clough, as already mentioned [6], used two beams interconnected by rigid links and springs in order to obtain a more realistic model of a ship with a double bottom girder capable of independent local deformations.

The structure of a tanker has also been approximated by a model of elastically connected parallel beams [25]. Each beam represents the side shells and the longitudinal bulkheads,

and the connecting springs represent the transverse bulkheads and the transverse rings.

Hylarides [26] suggests the finite element method as an effective means of taking into account the spatial composition of the hull, providing therefore a much more realistic three-dimensional representation of a ship structure. With the finite element technique, a complex structure like a ship's hull is considered as a composition of elements for which the stress-strain relations can easily be formulated. As a result, the imperfections of the beam method essentially caused by considering the hull as a one-dimensional vibrating beam are eliminated. The author also claims that their method also takes implicitly into account shear lag effects, local vibrations and coupling between horizontal and torsional vibrations.

Based on the finite element technique, a computer program called DASH (Dynamic Analysis of Ship Hulls) has been lately developed at the Netherlands Ship Model Basin [27].

Van Gunsteren [12] compares the beam theory with the finite element technique and points out some of the disadvantages of the latter, mainly from the computational point of view. In fact, the finite element method needs three-dimensional information about stiffness, wave loads and added mass, which is difficult to obtain. For lower modes of vibration the finite element technique only gives slightly more accurate results but requires considerable computing time and this may be its single important disadvantage.

From the review of the existing models we conclude that the beam theory provides a perfectly acceptable representation of a ship structure, at least for the lower modes of vibration, which we consider to be the only relevant area for the slamming response analysis. The important aspect that influences the response is the frequency associated with a particular mode, rather than the mode shape itself, since the slamming loads have a definite frequency content that is close to the value of the first mode natural frequency [28].

The beam theory is therefore chosen since it is capable of conveniently representing the ship's structure and also because of the important advantage of providing a straightforward mathematical formulation for the analysis, if certain simplifications can be taken into consideration.

2. MATHEMATICAL FORMULATION

The damped vertical response of a ship's hull to transient forces, assuming it behaves like a free-free nonuniform beam of length L, is governed by the following system of partial differential equations [29]:

$$\mu(x)\,\frac{\partial^2 y}{\partial t^2} + c(x)\,\frac{\partial y}{\partial t} + \frac{\partial V(x,t)}{\partial x} = P(x,t) \tag{7}$$

$$\frac{\partial M(x,t)}{\partial x} = V(x,t) + I_r(x)\,\frac{\partial^2 \gamma}{\partial t^2} \tag{8}$$

$$M(x,t) = EI(x)\,\frac{\partial \gamma(x,t)}{\partial x} \tag{9}$$

$$\frac{\partial y}{\partial x} = - \frac{V(x,t)}{KAG(x)} + \gamma(x,t) \tag{10}$$

x = distance in the longitudinal direction measured from the origin of the coordinate system

t = time variable

y = vertical elastic deflection, normal to x

μ = effective mass per unit length, or ship's mass per unit length $m'(x)$ plus added mass per unit length $m(x)$. In this case we take the added mass as a function of space alone based on the ship's calm water waterline sections (high-frequency limit)

c = damping coefficient per unit length

V = shear force in y direction

P = total force per unit length due to the ship-wave interaction, given by (6)

M = bending moment

I_r = mass moment of inertia of hull per unit length with respect to an axis normal to the x-y plane

γ = component of the slope of y due to bending only

EI = bending rigidity, where E is the modulus of elasticity and I the sectional area moment of inertia

KAG = shear rigidity, where K is the ratio of the average shear stress to the shear stress at the neutral axis under vertical load, A is the section area and G the shear modulus.

The ship is assumed to have free ends so that the boundary conditions are:

$$V(-L/2,t) = V(L/2,t) = M(-L/2,t) = M(L/2,t) = 0 \qquad (11)$$

This system of partial differential equations with its boundary conditions has been solved by several numerical methods that in general transform them into a set of implicit finite-difference equations which can be solved by computer [4]. The parameters and variables are then treated as discrete rather than distributed. One of the difficulties of the method which becomes one of the sources of inaccuracies of the beam theory as already pointed out is that it requires an evaluation of the bending and shear rigidity distributions, for which an exact procedure is not really available. Besides this fact, the solution of a system of partial differential equations, which to be of practical use must use a computer, involves also approximated numerical methods that are far from accurate. However, it happens that the formulation may be considerably simplified, as sketched below, in fact leading to a single constant coefficients linear second order differential equation not involving the parameters $EI(x)$ and $KAG(x)$, if we neglect the term involving rotary inertia I_r.

The Report of the Committee on Vibration of the Third International Ship Structures Congress [22] specifically states that investigations showed that there is little effect of the rotary inertia on the natural frequency of the vertical

vibration of destroyers. Van Gunsteren [12] neglects this term in his analysis, saying that for prismatic bars its influence on the natural frequency is a quarter of the influence of shear, which is already of small influence in comparison with bending.

Accepting then to neglect the influence of rotary inertia, the analysis further shows that the dynamic behavior of the beam can be treated in terms of series of responses in each of its normal modes i, which retain the important property of orthogonality with respect to the effective mass per unit length [30]:

$$\int_{-L/2}^{L/2} \mu(x) X_i(x) X_j(x)\, dx = 0 \tag{12}$$

Here $X_i(x)$ is the normal mode function in arbitrary dimensionless units, and it simply represents a pattern of relative displacements along the length of the beam for a particular mode i.

A generalized coordinate with the dimensions of length $q_i(t)$ is used to define the displacement time history of the system in its i^{th} normal mode. Then the motion in a particular mode i is given by multiplying $q_i(t)$ by the dimensionless normal mode function $X_i(x)$, and the total response is finally given by summing the contributions from all the modes:

$$y(x,t) = \sum_{i=1}^{\infty} q_i(t) X_i(x) \tag{13}$$

Similarly, we can represent M(x,t) and V(x,t) as the product of $q_i(t)$ by a spatial weighting function $M_i(x)$ or $V_i(x)$, respectively, and the form of these functions will be determined from the analysis.

$$M(x,t) = \sum_{i=1}^{\infty} q_i(t) M_i(x) \tag{14}$$

$$V(x,t) = \sum_{i=1}^{\infty} q_i(t) V_i(x) \tag{15}$$

We assume that P(x,t) can be written in the following series form [30]:

$$P(x,t) = \sum_{i=1}^{\infty} \frac{\mu(x) Q_i(t) X_i(x)}{\int_{-L/2}^{L/2} \mu(x) X_i^2(x)\,dx} \tag{16}$$

Multiplying both sides of (16) by $X_i(x)$ and integrating over the ship's length, the orthogonality property (12) leads to an explicit form for the function $Q_i(t)$:

$$Q_i(t) = \int_{-L/2}^{L/2} P(x,t) X_i(x)\,dx \tag{17}$$

Neglecting the term involving I_r and substituting in the equations (7) to (10) (where (9) and (10) may be readily combined) the series representations for y(x,t), M(x,t), V(x,t) and P(x,t), (13) to (16), we get the three following equations (for simplification of notation we drop here the functional dependencies on time and space variables, and we use dots to denote differentiations with respect to time):

$$\sum_{i=1}^{\infty}\left(\mu\ddot{q}_iX_i + c\dot{q}_iX_i + q_i\frac{dV_i}{dx} - \frac{\mu Q_iX_i}{\int_{-L/2}^{L/2}\mu X_i^2dx}\right) = 0 \tag{18}$$

$$\sum_{i=1}^{\infty}\left(q_iV_i - q_i\frac{dM_i}{dx}\right) = 0 \tag{19}$$

$$\sum_{i=1}^{\infty}\left[q_i\frac{d^2X_i}{dx^2} + q_i\frac{d}{dx}\left(\frac{V_i}{KAG}\right) - \frac{q_iM_i}{EI}\right] = 0 \tag{20}$$

These equations are satisfied if each term in the summation is set equal to zero. Combining the resulting equations we get

$$\mu\ddot{q}_iX_i + c\dot{q}_iX_i + q_i\frac{d^2}{dx^2}\left[EI\frac{d^2X_i}{dx^2} + EI\frac{d}{dx}\left(\frac{V}{KAG}\right)\right] = \frac{\mu Q_iX_i}{\int_{-L/2}^{L/2}\mu X_i^2dx} \tag{21}$$

If we consider the free motion of the beam with no forcing function acting, the right-hand side of (21) becomes zero and for a normal mode, after rearranging the equation we get

$$\frac{\ddot{q}_i + (c/\mu)\dot{q}_i}{q_i} = -\frac{1}{\mu X_i}\frac{d^2}{dx^2}\left[EI\frac{d^2X_i}{dx^2} + EI\frac{d}{dx}\left(\frac{V}{KAG}\right)\right] = -\frac{1}{\mu X_i}\frac{d^2M_i}{dx^2} \tag{22}$$

Since here the left hand side is just a function of time and the right hand side just a function of space, we conclude that both must be equal to a constant $-w_i^2$, where w_i is the

natural frequency of the i^{th} mode. This leads to

$$\ddot{q}_i + (c/\mu)\dot{q}_i + w_i^2 q_i = 0 \tag{23}$$

$$\frac{d^2M_i}{dx^2} = \frac{d^2}{dx^2}\left[EI\,\frac{d^2X_i}{dx^2} + EI\,\frac{d}{dx}\left(\frac{V}{KAG}\right)\right] = \mu w_i^2 X_i \tag{24}$$

Integrating (24) along the length of the beam we get

$$M_i = \int_{-L/2}^{x}\int_{-L/2}^{x} \mu w_i^2 X_i\,dxdx \tag{25}$$

The following integral gives a more convenient representation for M_i.

$$M_i = w_i^2 \int_{-L/2}^{x} (x-s)\,\mu(s)X_i(s)\,ds \tag{26}$$

Substituting (24) in (21), multiplying both sides by X_i and taking the space integral of both sides from -L/2 to L/2, we get, using again the orthogonality principle (12),

$$\bar{\mu}_i\ddot{q}_i + \bar{c}_i\dot{q}_i + \bar{k}_i q_i = Q_i \tag{27}$$

where $\bar{\mu}_i$, which we can call the generalized mass, is defined by

$$\bar{\mu}_i = \int_{-L/2}^{L/2} \mu X_i^2\,dx \tag{28}$$

and $\bar{c}_i$, the generalized damping is

$$\bar{c}_i = \int_{-L/2}^{L/2} cX_i^2\,dx \tag{29}$$

and $\bar{k}_i$, the generalized spring constant is

$$\bar{k}_i = w_i^2\bar{\mu}_i \tag{30}$$

Since the effective mass per unit length μ is only a function of x it follows that $\bar{\mu}_i$ is a constant for a particular mode i and it has dimensions of mass. The generalized damping $\bar{c}_i$, which will be discussed in more detail in the next section, is for similar reasons also a constant and so the coefficient of $\dot{q}_i$ in (27) is constant. Since w_i or the natural frequency of the i^{th} mode has a certain fixed value, we conclude that (27) is a simple constant coefficients linear second order differential equation where the unknown is $q_i(t)$ and the forcing function is $Q_i(t)$. The solution of such an equation is simple if initial conditions are given. At this point we can assume that at time t=0 the beam is at rest so that $q_i(0) = \dot{q}_i(0) = 0$. The solution is then given in a closed form as

$$q_i(t) = \int_0^t \frac{Q_i(\tau)}{\lambda_i \bar{\mu}_i} \exp\left[-(\bar{c}_i/2\bar{\mu}_i)(t-\tau)\right] \sin\left[\lambda_i(t-\tau)\right] d\tau \tag{31}$$

$$\lambda_i = \sqrt{w_i^2 - (1/4)(\bar{c}_i/\bar{\mu}_i)^2} \tag{32}$$

Knowing $q_i(t)$ as well as the normal mode shapes and natural frequencies, it is possible to compute $M_i(x)$ from (26) and finally obtain the bending moment from (14) for any location x along the ship.

The integral definition of $M_i(x)$ given in (26) can be interpreted as if we considered the main contribution to the bending moment to be a result of the inertial loads distribution along the hull due to the vibratory deflection, including the fluid inertial force associated with the added mass.

The differential equation (27) and its closed form solution (31) represent, then, the analytical solution to the ship structural model and now the great advantage of its final computational simplicity becomes more evident. The analysis will be complete after giving a more rigorous definition to the several parameters involved, particularly the added mass and the damping coefficient.

3. DEFINITION OF PARAMETERS

The solution for the transient response of the ship structure to a bottom impact slamming load, as given by the normal approach to the beam theory, requires the knowledge of a certain number of parameters, namely ship mass and added mass distribution, damping coefficient and normal mode shapes and natural frequencies.

The ship's mass or weight distribution may be determined through classical methods from a knowledge of its structural main characteristics and internal arrangements. In general, this calculation is part of the design process and we may consider it here as a readily available piece of information.

The added mass distribution is a more involved problem and the approach followed here is heavily based on the one adopted by the Computer Seakeeping Program developed at M.I.T. [31]. Here the computation of the sectional added mass is based on the hydrodynamic problem of determining it for an infinite length cylinder floating at an infinite depth of

water and oscillating vertically with a small harmonic motion, the viscous and end effects being neglected. The sections are defined as Lewis forms with main parameters area, draft and half beam, and the numerical procedure for the actual computation of the added mass is the one due to Grim. The two main restrictions of this algorithm are taken into account in the sense that it cannot be successfully used either for very small sectional areas or for bulbous sections with very large sectional areas. The added mass resulting from this method is frequency dependent (frequency of encounter), which should be taken into account when computing the forcing function P(x,t) as given by (6).

When seeking the coefficient of $\ddot{q}_i(t)$ in (27) the high frequency asymptotic limit of the added mass is used, since the ship's natural frequency is relatively high and therefore we can assume the added mass to have already reached its frequency independent asymptotic value.

In the next section describing the proposed general procedure it will be mentioned how the calculations will be carried out for specific numerical applications.

The problem of defining the damping associated with hull vibration is quite difficult since its mechanisms have not yet been completely explained.

In [22] ship hull damping is classified as a combination of the energy dissipation effects that occur not only in the

fluid medium but also within its own structure. The internal or structural damping is the result of such varied mechanisms as working and fraying of overlapping plates, motion of loose cargo in the hull and damping between rivets and structural joints. On the other hand, the hydrodynamic or external damping can result from energy losses with low frequency surface wave generation, water friction or losses in the boundary layer and high frequency acoustic energy dissipation.

Investigations have concluded that the hydrodynamic damping is less important than the internal damping by at least one order of magnitude but the contribution due to the generation of surface waves in general may be significant. Also, the internal material damping is negligible and cargo damping has not yet been really described. Experimental measurements give results for the damping coefficient that can be assumed to include the effects of the structure, cargo and minor hydrodynamic effects like water friction and the generation of pressure waves.

Goodman [11] uses data from Johnson and Ayling [32] giving in a graphical form the damping ratio due to effects other than the generation of surface waves and forward speed, as a function of natural frequency. In a discussion of the same paper [11], Belgova criticizes this approach, stating that the dependence of damping upon frequency should only be established from data for a particular mode at a time, since

the conditions for the behavior of the ship structure in different vibration modes are not the same.

In spite of its shortcomings the approach suggested by Goodman for the damping coefficient definition will be adopted, since it is to our knowledge the most recent work in which this problem is raised.

We will consider three main factors in the damping coefficient: first, a structural damping factor c_s/μ which includes also the minor hydrodynamic effects of water friction and generation of pressure waves, and which is only a function of the mode frequency; second, the generation of surface waves effect; and finally, a forward speed correction factor that takes the form $-U(dm/dx)$. Note that c_s and $-U(dm/dx)$ are given on a per-length basis.

At the relatively high ship's structural mode frequencies the effect of damping due to wave generation may be considered negligible as compared with the other mechanisms that are included in c_s/μ. In fact, as already mentioned, the structural damping may be about an order of magnitude more important than the hydrodynamic damping. Then we are left with the two contributions c_s/μ and $-U(dm/dx)$ as defined before, and using the definitions (28) and (29) we get

$$c = c_s - U\frac{dm}{dx} = \left(\frac{c_s}{\mu}\right)\mu - U\frac{dm}{dx} \tag{33}$$

$$\bar{c}_i = \int_{-L/2}^{L/2} \left[\left(\frac{c_s}{\mu}\right)\mu - U\frac{dm}{dx}\right] X_i^{\,2} dx \tag{34}$$

$$\bar{c}_i = \left(\frac{c_s}{\mu}\right)\bar{\mu}_i - U\int_{-L/2}^{L/2} \frac{dm}{dx} X_i^{\,2} dx \tag{35}$$

To complete the knowledge of the parameters required to solve completely the vibratory response of the ship, we finally need the normal mode shapes $X_i(x)$ and natural frequencies w_i.

Several procedures have been developed in order to compute $X_i(x)$ and w_i, like the Prohl-Myklestad digital method reported in [22] or the finite element program DASH already mentioned [27].

The literature reports mode vibration data for various types of ships under different load conditions, and so we can use it for similar designs if the computer methods are not readily available.

Summarizing this section we may say that the required parameters for the ship's transient response solution are obtained in the following way:

a) the ship mass distribution by the standard naval architecture procedure;

b) the added mass using a computer program applying the Lewis form concept in conjunction with the computation method of Grim;

c) the structural damping coefficient c_s/μ will be taken from a graphical representation giving its

relation to the natural frequency [11]; the generalized damping will be computed in accordance with (35);

d) the mode shape functions and natural frequencies will be either computed using available computer programs or derived from existing data for similar ships.

IV. GENERAL PROCEDURE

1. INTRODUCTION

Having discussed the theoretical aspects of the proposed method for the evaluation of a ship response to bottom impact slamming, we must now formulate it in a general procedure able to be conveniently applied to specific cases. This will mainly be a discussion of the sequence of operations to be performed, since the mathematical formulation has already been given. The method will be further illustrated by a numerical application described in Part V.

Before proceeding, a point to take into consideration is the description of the sea conditions for which the response is to be evaluated. The most realistic situation and the one more likely to give more useful information is the one of irregular seas characterized by a certain sea state or energy spectrum. However, the first step to be taken must concern the rather simplified situation of regular waves, from which a more general irregular-waves result can eventually be derived.

The problem of extending results from regular to irregular waves may be quite involved and several approaches may have to be tried. In any case, the possibility of a linear dependence on the amplitude of the waves should be studied, since if we admitted linearity to exist, the use of the superposition principle would be possible. In this case, we would consider

the seaway to be composed of many regular wave components, each having its own amplitude, length and direction of travel. Response values per foot of amplitude for several frequencies in conjunction with a sea energy spectrum would then lead the corresponding response spectrum. The acceptance of this approach has, however, to be verified in terms of the linearity of the response with the wave amplitude, which may not be obvious.

Another possible way, at least formally, would be to study the feasibility of treating the ship as a system of one degree of freedom acted by a driving force $Q_i(t)$, as suggested by equation (27). Then using spectral analysis concepts the knowledge of the frequency spectrum of the forcing function would lead to the frequency spectrum of the response, in this case $q_i(t)$. The problem would then be how to find the frequency spectrum of $q_i(t)$ or $P(x,t)$, and how to find from the spectrum of $q_i(t)$ the frequency spectrum of the bending moment, which is ultimately what we are interested in.

The present work will only deal with the response in regular waves of a given frequency and amplitude, and this will hopefully become the basis for the generalization of the method in order to handle the more realistic case of irregular waves.

An important point of the procedure is the computation of the ship motions in regular waves, which will be made using

the Seakeeping Program already mentioned [31]. Here the prediction of seakeeping performance is still limited to two modes of ship motions (pitching and heaving) for head long-crested seas. The theory adopted in the report is essentially the linear strip theory of Korvin-Kroukovsky where surge, sway, yaw and roll have been neglected.

The consideration of only two degrees of freedom in head seas is acceptable for the study of slamming since these conditions lead to the most severe bottom impacts [8].

2. DESCRIPTION OF STEPS

The steps involved in the computational procedure are given below. For ease of reference they are denoted by the capital letters (A) to (G).

(A). Ship description

The following are assumed to be given: main dimensions and characteristics, mass distribution, natural mode shapes and frequencies, table of offsets or ship's lines and Bonjean curves (sectional area curves).

The structural damping coefficient c_s/μ is taken from Figure 2 in [11] for the different natural frequencies.

The threshold velocity v* comes from (1b) and the necessary conditions for slamming are found from (1a).

(B) Regular waves description (frequency)

The frequency w is selected within the frequency range of a sea energy spectrum of particular interest.

The frequency of encounter w_e is determined from (3).

(C) Added mass

The frequency-dependent added mass is calculated using a computer program we call ADMASS which is based on [31].

For each station the computations are repeated for several drafts and for each section the required inputs are the frequency of encounter, the draft, and the half breadth and sectional area known from the ship's lines and Bonjean curves. Then for each station we shall have for a certain frequency of encounter a set of values of the added mass, each corresponding to a particular draft, so that a curve may be drawn representing the variation of m with T (Figure 1).

The high frequency limit of m is evaluated using the same computer program ADMASS, for every station and for the inputs half breadth, draft and sectional area at the calm water waterline. The frequency input is increased so that a limiting asymptotic value may be obtained. This high frequency limit added mass will then be taken as being only a function of space or m(x), and it will be summed to the ship's mass distribution m'(x) in order to get the effective mass μ(x). The procedure is sketched in Figure 2.

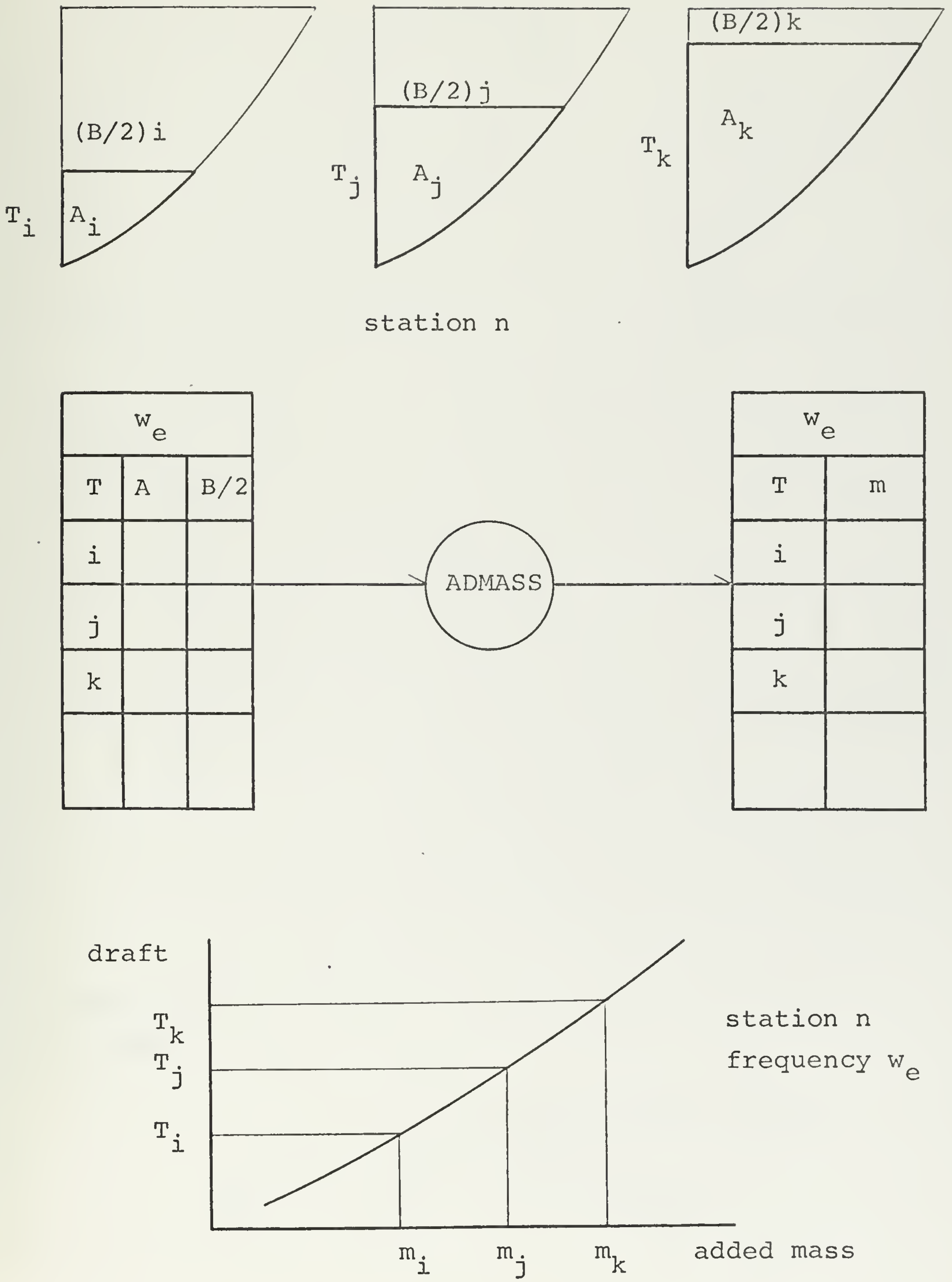

Figure 1 Added mass as a function of the draft for station n

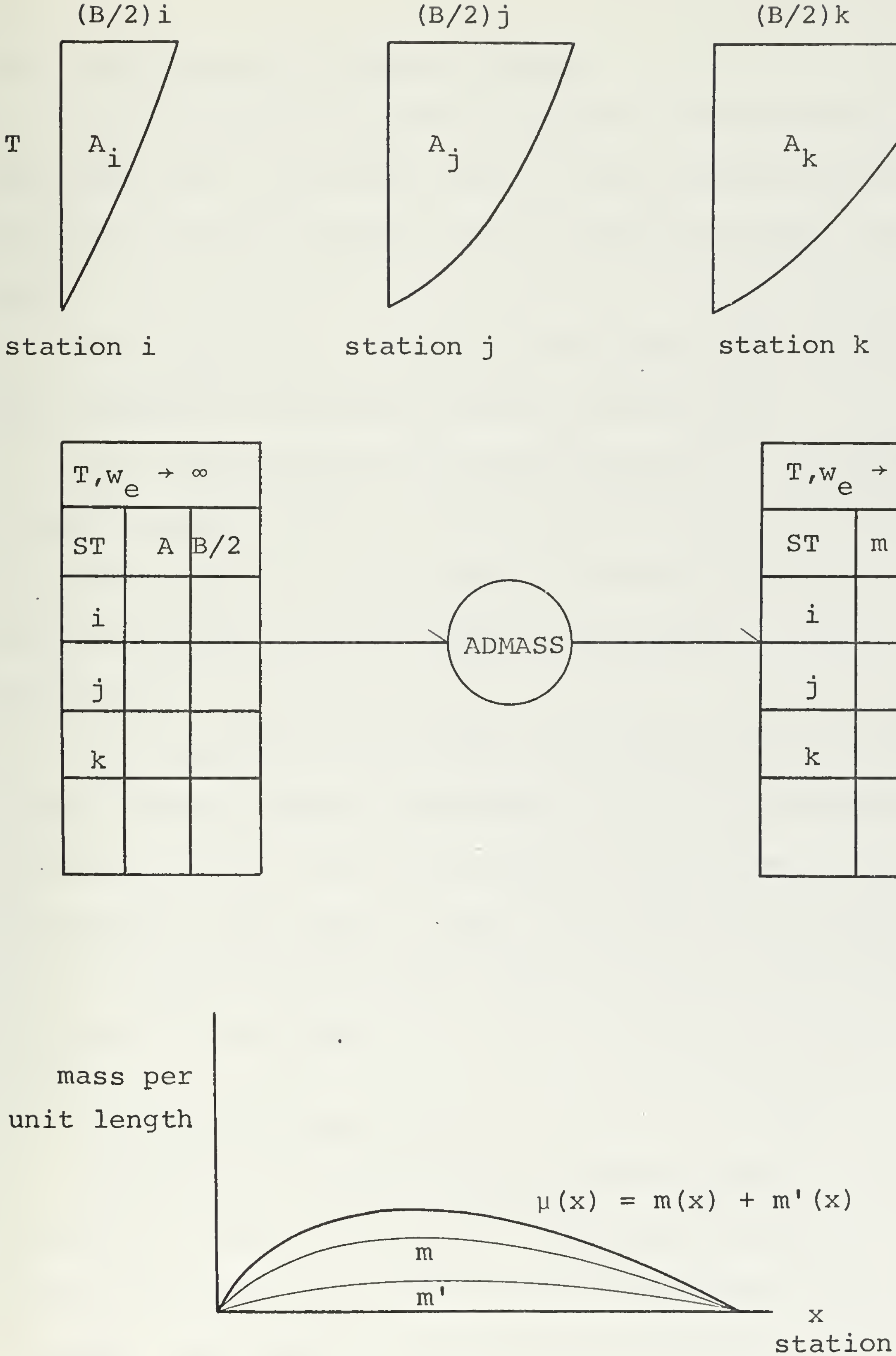

Figure 2 Effective mass computation

(D) Ship motions in regular waves

The Seakeeping Program [31] gives for each ship station the amplitudes and phase angles of the relative motion $z(x,t)$ and the relative velocity $\dot{z}(x,t)$ between the hull and the wave.

The frequency of the relative motions and velocities is the frequency of encounter known from step (B).

The wave frequency also known from (B) is a required input for the program, as well as a set of offsets which may be taken from the ship's lines. If the offsets are not available the program may use simply the principal ship's characteristics.

For a unit wave amplitude the program gives non-dimensional motion response amplitudes, so that an amplitude may be selected such that the conditions for slamming (1) are satisfied. This means we know the relative motions and velocities (amplitude and phase angle) at every ship station when slamming is occurring.

(E) Loading function $P(x,t)$

Knowing the relative motion and relative velocity at each station in terms of frequency, amplitude and phase, and since these are sinusoidal functions we can generate their time history. This means we can have for different equally spaced instants of time, which we must select, a value of the relative motion and the relative velocity at each station.

The relative motion in conjunction with the draft will give the instantaneous draft $T(t)$, or the immersed portion of the hull at a certain station and for a particular time instant.

From (C) we can take the value of the added mass m for each instantaneous draft since we have a graphical representation of the variation of m with T. Similarly, from the Bonjean curves we can take the value of the sectional area for each station and for each instantaneous draft.

We finally have for each station and for equally spaced instants of time the values of the relative velocity $\dot{z}(x,t)$, the added mass $m(x,t)$ and the sectional area $A(x,t)$. After performing the product of the added mass by the relative velocity ($m\dot{z}$) for each time and station, numerical differentiations are required to finally obtain $P(x,t)$ as given by equation (6).

The procedure is sketched in Figure 3, where I denotes the operation of finding the instantaneous immersion and II the solution of (6).

(F) $\underline{Q_i(t)}$, $\underline{M_i(0)}$ <u>and generalized coefficients</u>

The computation of $Q_i(t)$, $M_i(o)$, $\overline{\mu}_i$ and $\overline{k}_i$ follow immediately from equations (17), (26), (28) and (30) respectively, the first three involving an integration.

The computation of $\overline{c}_i$, which involves an integration and a differentiation, is now possible since we know all the terms in (35).

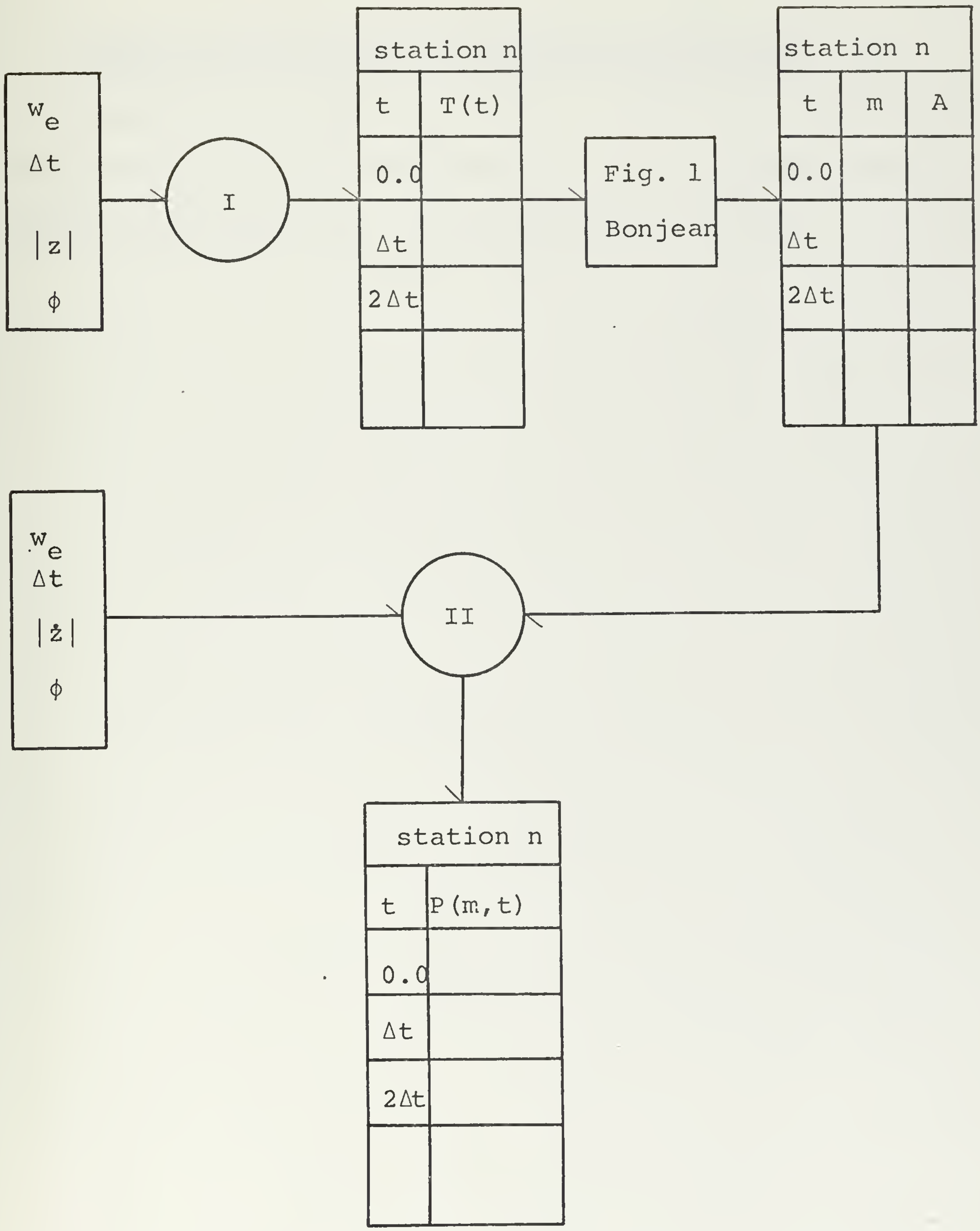

Figure 3 Computation of the loading function for station n

(G) Closed solution of the governing equation (31) and computation of the midships bending moment (14)

Equations (31) and (14) can be solved in one step, leading to the final result of interest $M(0,t)$, or the midships bending moment time history for a particular type of wave.

V. NUMERICAL EXAMPLE: APPLICATION TO A MARINER SHIP

The main results of a numerical application to the MARINER will be given in this section.

We should notice it is not worthwhile to publish in detail all the calculations, since most of them are of no particular significance. Then, besides reporting the more important results, the decisions that were taken along the procedure will also be discussed and justified.

The MARINER type was chosen since there are many of these ships in operation today, and also in has been extensively studied in the past. Examples are the already-mentioned works by Ochi [8] and Lewison[17].

1. GENERAL PROCEDURE AND MAIN RESULTS

(a) Ship description

The MARINER principal characteristics are listed in Table 1.

The selected speed was 16 knots. For the MARINER the speed range of 15 to 20 knots has been considered as critical for slamming [33], and this is the reason why the value of 16 knots was chosen.

The weight distribution is given in Figure 4.

The first mode natural frequency is given by Kaplan [7] for a MARINER, as $w_1 = 9.42$ rad/sec. This author takes the first mode shape to be a simple parabolic curve.

Table I

General Characteristics of the MARINER

Length overall, ft-in	563-7.75
Length between perpendiculars, ft-in	528-0
Beam, maximum, molded, ft-in	76-0
Depth, main deck at side, ft-in	44-6
Draft, ft-in	27-0
Displacement, ton	18,674
Block coefficient	0.6125
Prismatic coefficient	0.6246
Midshipsection coefficient	0.9807
Number of stations	20
Station spacing, ft-in	26-0

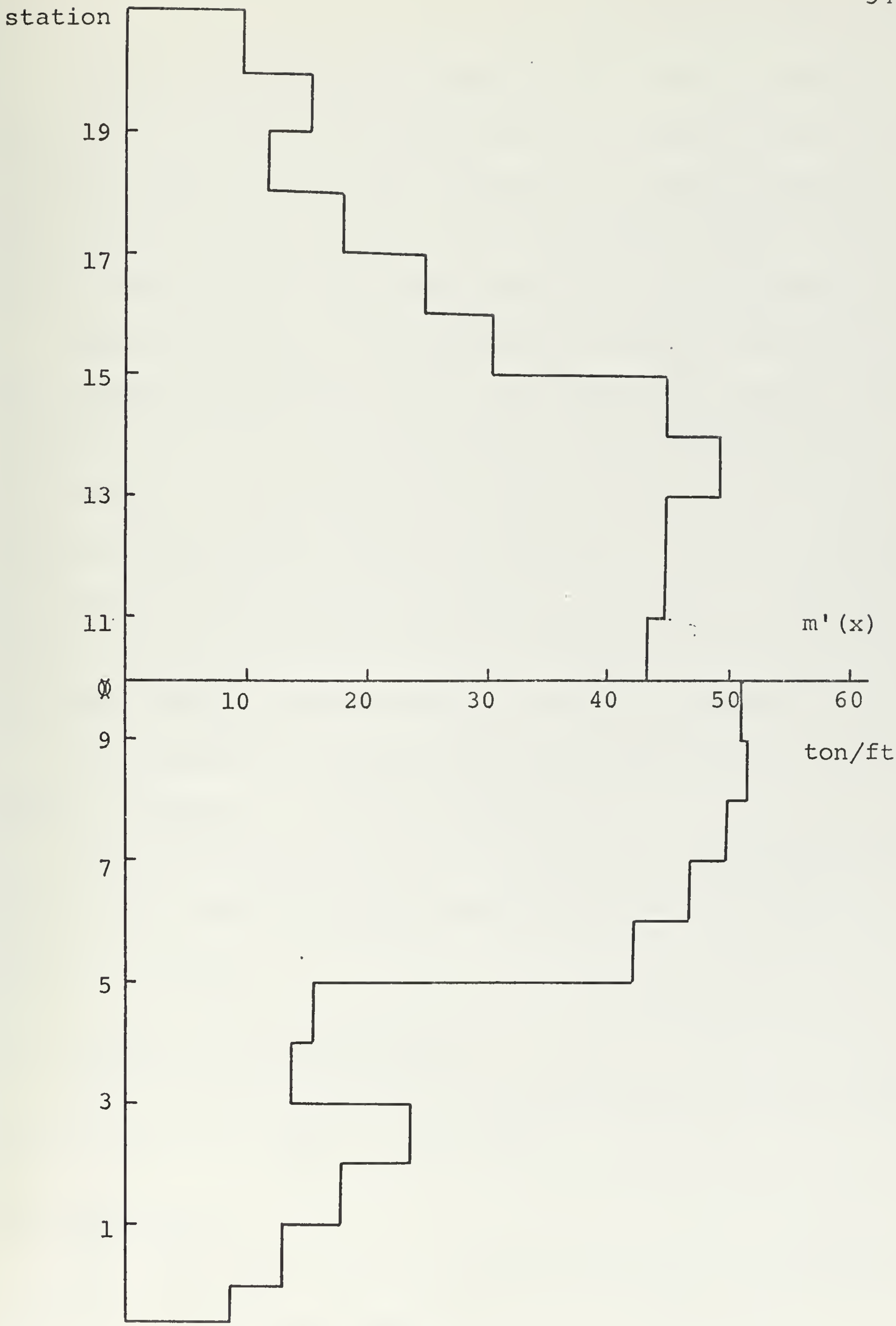

Figure 4 Mariner weight distribution

Data for the vibration modes 2 and 3 was not readily available, so that we decided to use the results found for a similar ship. Since these modes have a rather secondary contribution to the response, at least when compared with mode 1, the imprecisions introduced should not be very significant.

The report on the NS SAVANNAH [24] became the source of the required information about mode shapes and natural frequencies. In fact, the NS SAVANNAH and the MARINER types are similar in their principal dimensions, as shown in [34]. Besides, the first mode natural frequency for the NS SAVANNAH is given as 9.498 rad/sec, which is very close to the figure of 9.42 rad/sec mentioned above. For these reasons we decided to adopt the data given in [24], as shown in Figure 5.

The structural damping coefficient c_s/μ for each mode natural frequency is given in Table II.

The threshold velocity v* is 12 ft/sec, so that the conditions for slamming may be written as $|z| \geq 27$ ft and $|\dot{z}| \geq 12$ ft/sec.

(B) Regular waves description

The selected frequencies and correspondent frequencies of encounter are given in Table III.

(C) Added mass

The added mass per unit length plotted as a function of the draft for the particular case of $w = 0.8$ rad/sec

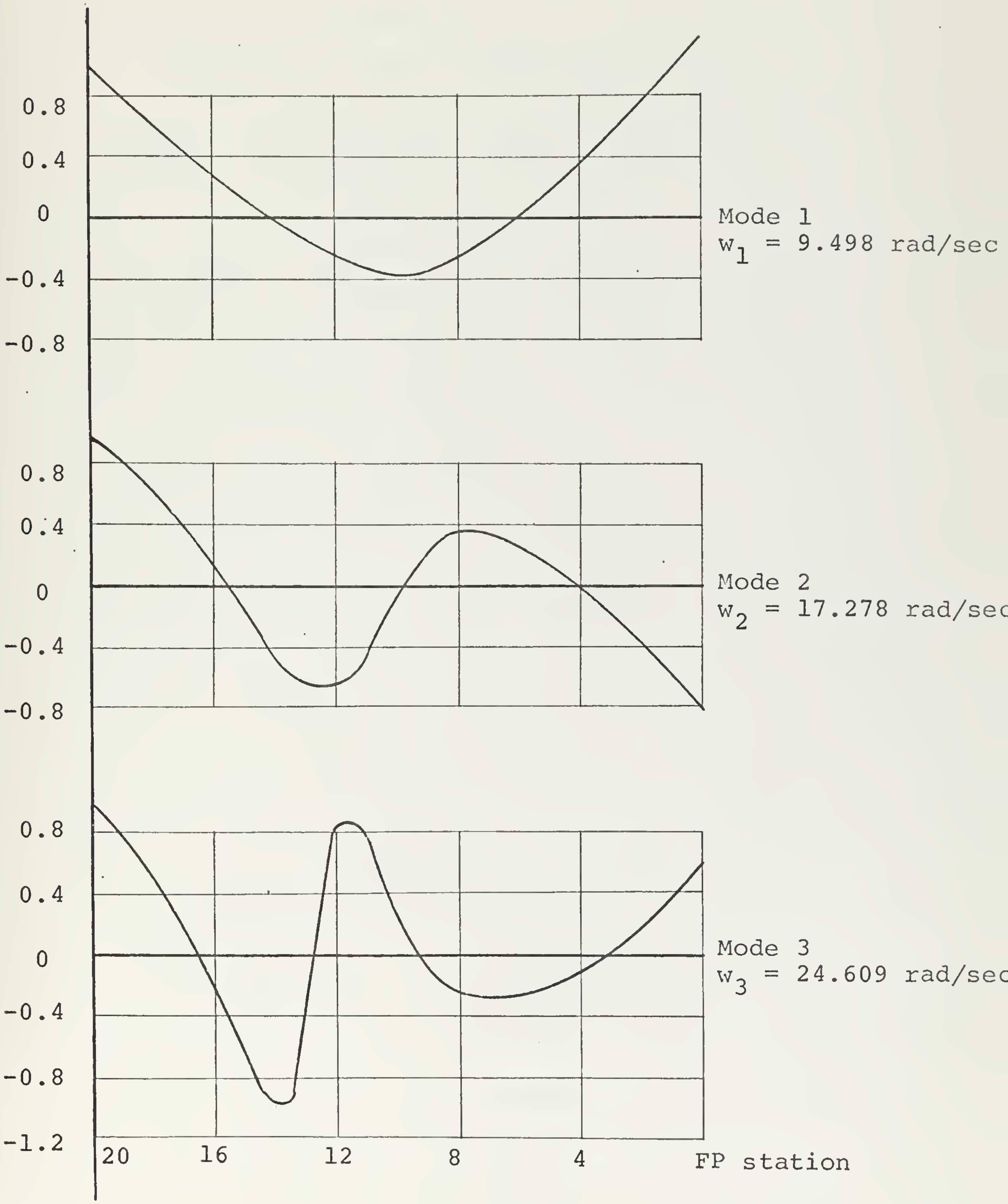

Figure 5 Mariner mode shapes

Table II

Structural Damping Coefficient

mode	w_i (rad/sec)	c_s/μ (/sec)
1	9.498	0.055
2	17.278	0.130
3	24.609	0.260

Table III

Regular Waves Frequencies

w (rad/sec)	w_e (rad/sec)
0.2	0.2336
0.4	0.5344
0.6	0.9024
0.8	1.3375

(w_e = 1.3375 rad/sec) is given in Figure 6 for a selected number of stations.

The high frequency limit of the added mass, as used to compute the effective mass, is plotted in Figure 7.

(D) Ship motions in regular waves

A wave amplitude of 15 ft was found to satisfy the slamming requirements defined in (A). To illustrate this fact we give in Table IV the relative motion for the first seven stations, for the wave frequency of 0.4 rad/sec (wave amplitude 15 ft). We can see that the requirements $|z| \geq 27$ ft and $|\dot{z}| \geq 12$ ft/sec are both met for Stations 1 through 4. For the remaining stations the amplitudes of relative motion and velocity decrease, until they reach a minimum at Station 20.

(E) Loading function P(x,t)

First a simple computer program generates the values of the immersions at each of the 20 stations, for time intervals of 0.2 seconds starting at t = 0.0 seconds. The input is the frequency w_e plus the relative motion amplitudes and phase angles as obtained from (D).

The computer essentially prints out the values of a sinusoidal function of time to which it adds the ship's draft

$$z = |z| \sin(w_e t + \emptyset) + 27$$

for t_1=0.0, t_2=0.2, t_3=0.4, etc.

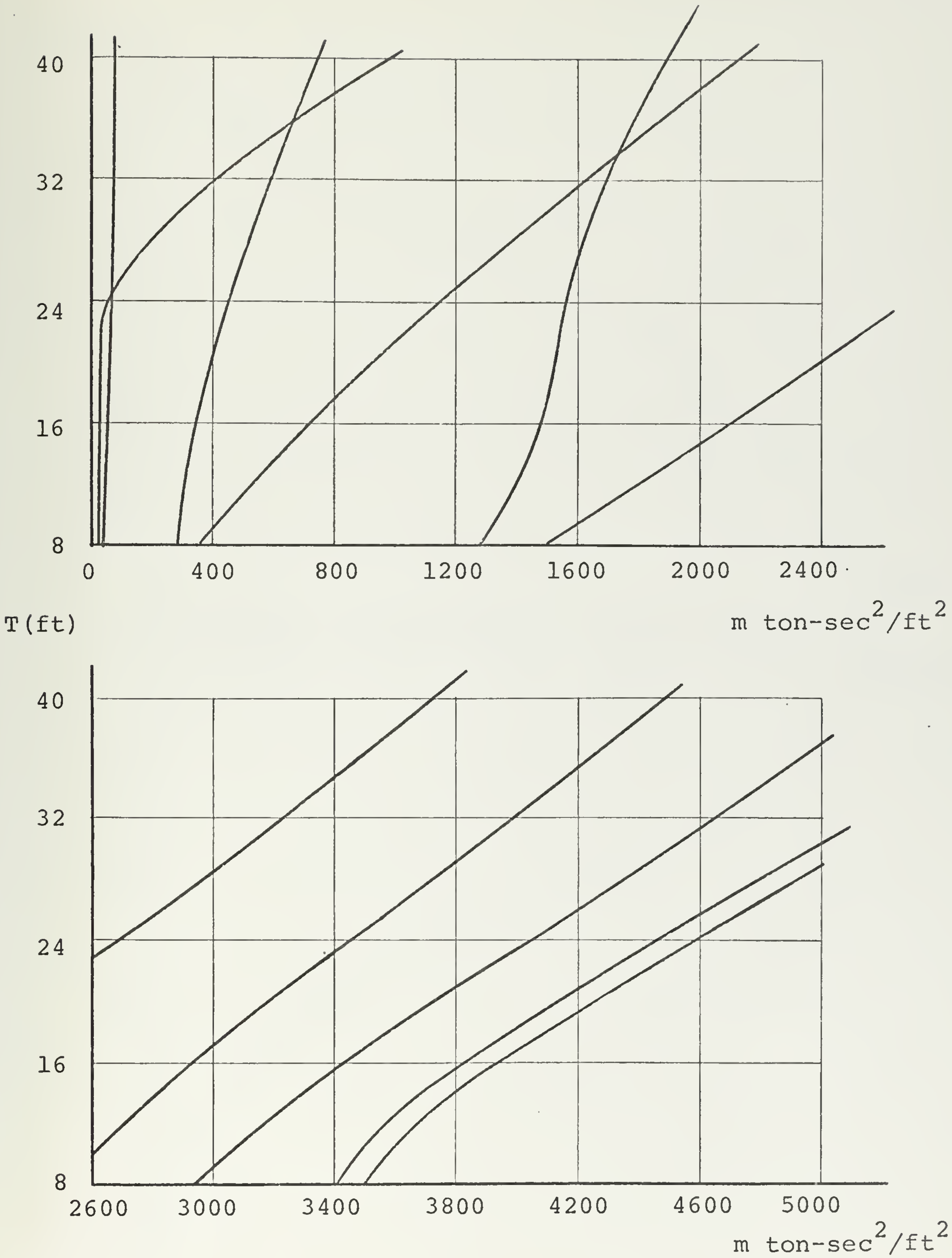

Figure 6 Added mass as a function of draft (w_e = 1.3375 rad/sec)

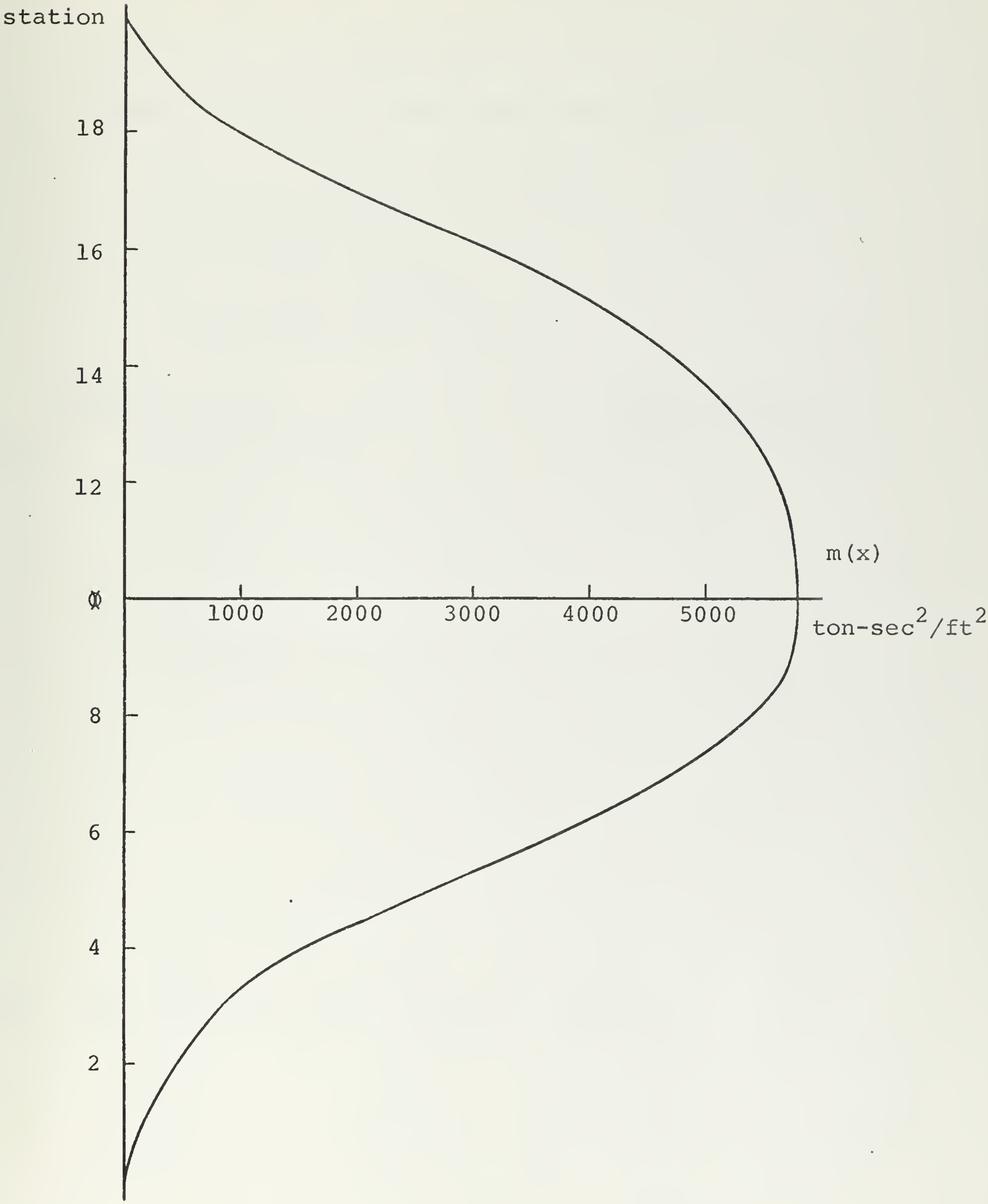

Figure 7 High frequency limit of the added mass

Table IV

Sample of Relative Motions and Velocities

V = 16 knots (27.0222 ft/sec)

WL/LBP = 2.4297

w = 0.4000 rad/sec

w_e = 0.5344 rad/sec

wave amplitude = 15 ft

Station	Relative Motion Amplitude (ft)	Relative Velocity Amplitude (ft/sec)
1	39.08	20.88
2	35.29	18.86
3	31.61	16.89
4	28.04	14.98
5	24.61<27	13.15
6	21.34<27	11.40<12
7	18.24<27	9.75<12

Table V

Mode	$\bar{\mu}_i$ (ton-sec^2/ft)	$\bar{c}_i$ (ton-sec/ft^2)	$\bar{k}_i$ (ton/ft)	$M_i(0)$ (ton)
1	362.45	21.377	32.7×10^3	$1,209 \times 10^3$
2	403.10	54.909	121.0×10^3	-679×10^3
3	754.09	200.333	450×10^3	$-1,568 \times 10^3$

This first printout gives the instantaneous immersions and for each of these we must tabulate the added mass and the sectional area.

A second computer program then performs the numerical differentiations and other operations required to obtain $P(x,t)$ as defined in equation (6). The differentiation subroutine uses a Lagrangian interpolation polynomial of degree 4 relevant to five equidistantly-spaced argument values. The input includes the frequency w_e, the relative velocity amplitudes and phase angles for the 20 stations, and the values of added mass and sectional area tabulated as explained before.

These two computer programs are called I and II respectively, in Figure 3, where the procedure is sketched.

(F) <u>$Q_i(t)$, $M_i(0)$ and generalized coefficients</u>

The same computer program II described before uses an integration subroutine to compute $Q_i(t)$. In addition to the defined input it must also read the mode shapes $X_i(x)$.

A third computer program which we call III is just a combination of numerical differentiation and integration subroutines, and it gives the values of $\overline{\mu}_i$, $\overline{c}_i$, $\overline{k}_i$ and $M_i(0)$ listed in Table V.

The integration algorithm follows Sympson's Rule together with Newton's 3/8 rule.

(G) <u>Closed solution of the governing equation (31) and computation of the midships bending moment (14)</u>

A computer program which we call IV solves equations (31) and (14). The numerical integration method is the one already described, and the input includes the results of step (F) along with the mode shapes $X_i(X)$.

Obviously II, III and IV may be combined in a single computer program as will be referred later.

The final results we were seeking, or the midship bending moment time history for the four input regular wave frequencies, are given in Figures 8 to 11.

2. DISCUSSION

The midship bending moment response to slamming we obtained is a time-varying function that presents a periodicity similar to the one of the waves, as would be expected. This is particularly evident for wave frequencies of 0.6 rad/sec and 0.8 rad/sec, where the plotted time histories are given for well over a period (Figures 10 and 11).

Referring to Figures 8 through 11 we may conclude the following:

a) For w=0.4 rad/sec the bending moment varies between -70×10^3 ft-ton and zero.

b) For w=0.4 rad/sec it varies between approximately -160×10^3 ft-ton and 130×10^3 ft-ton.

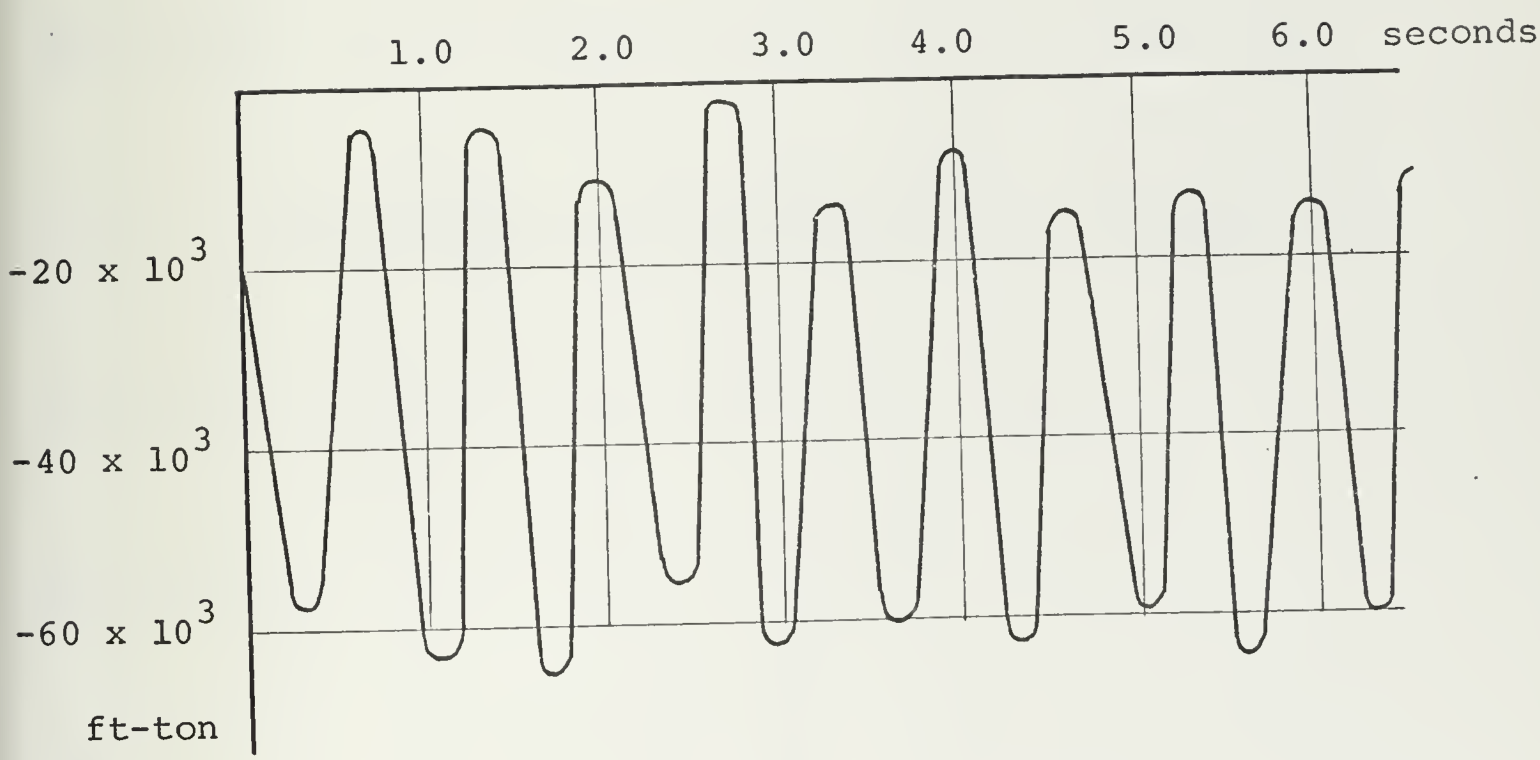

Figure 8 Midship bending moment for regular waves with w = 0.2 rad/sec

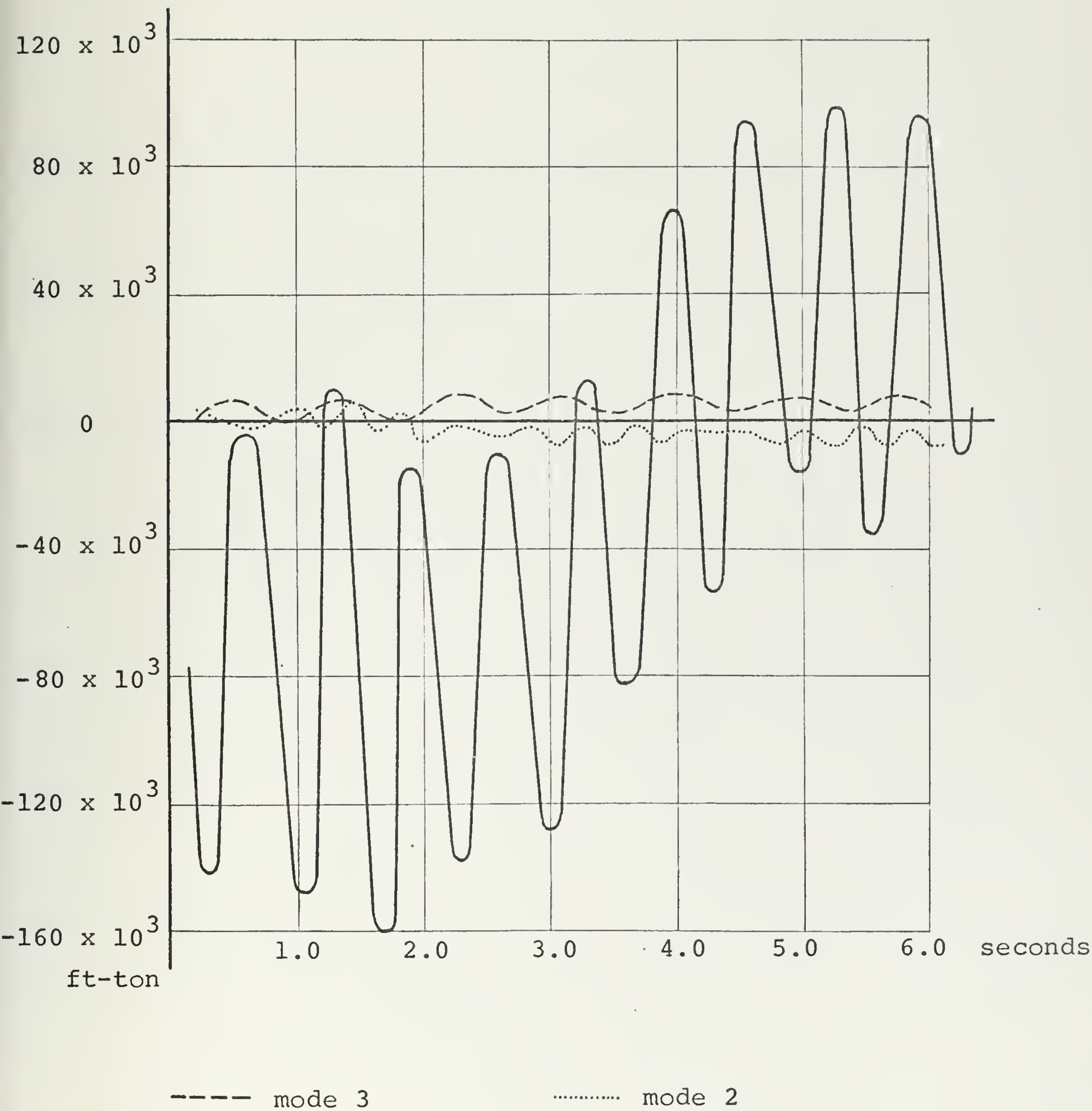

Figure 9 Midship bending moment for regular waves with w = 0.4 rad/sec. The contributions of modes 2 and 3 are also shown.

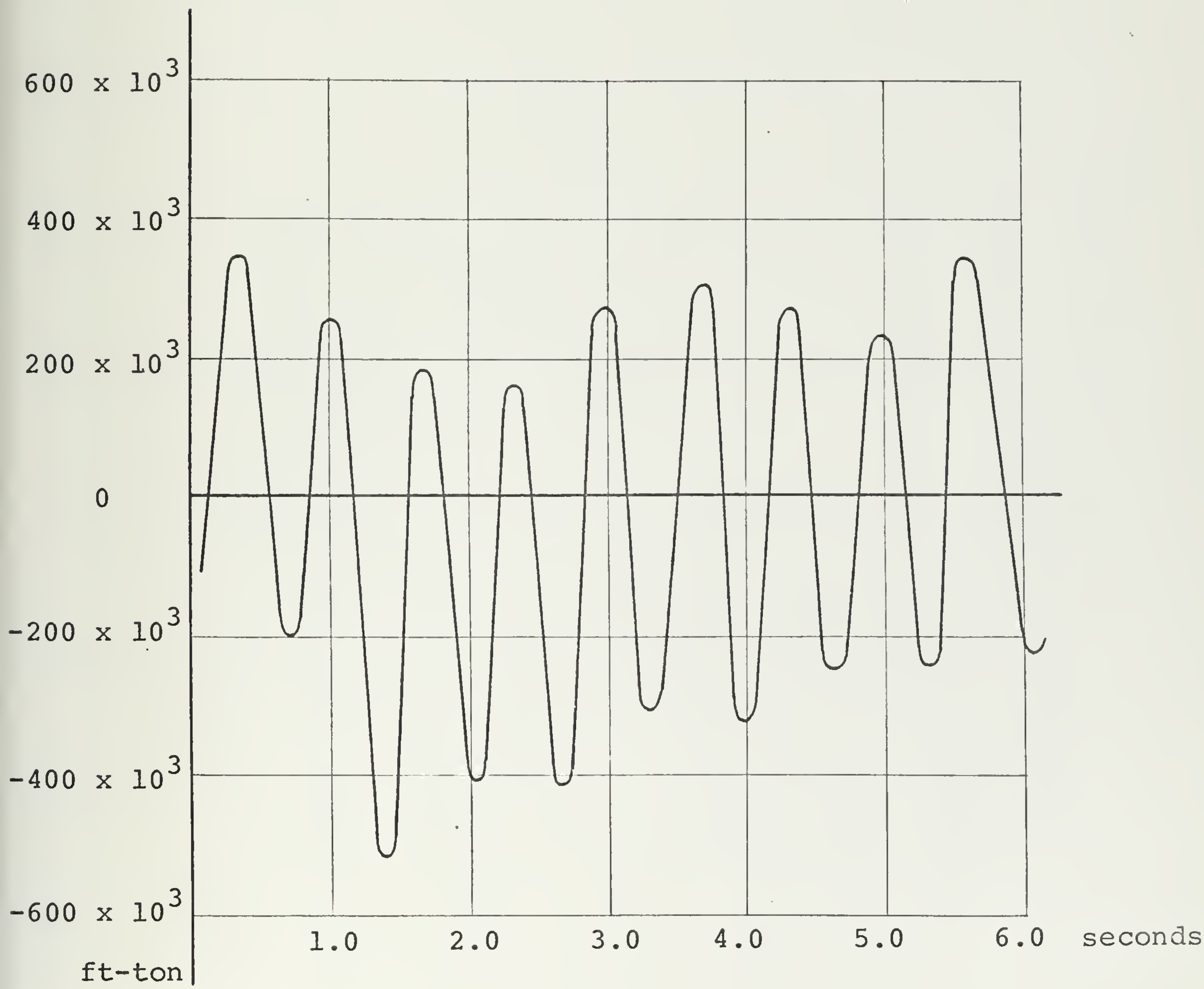

Figure 10 Midship bending moment for regular waves with $w = 0.6$ rad/sec

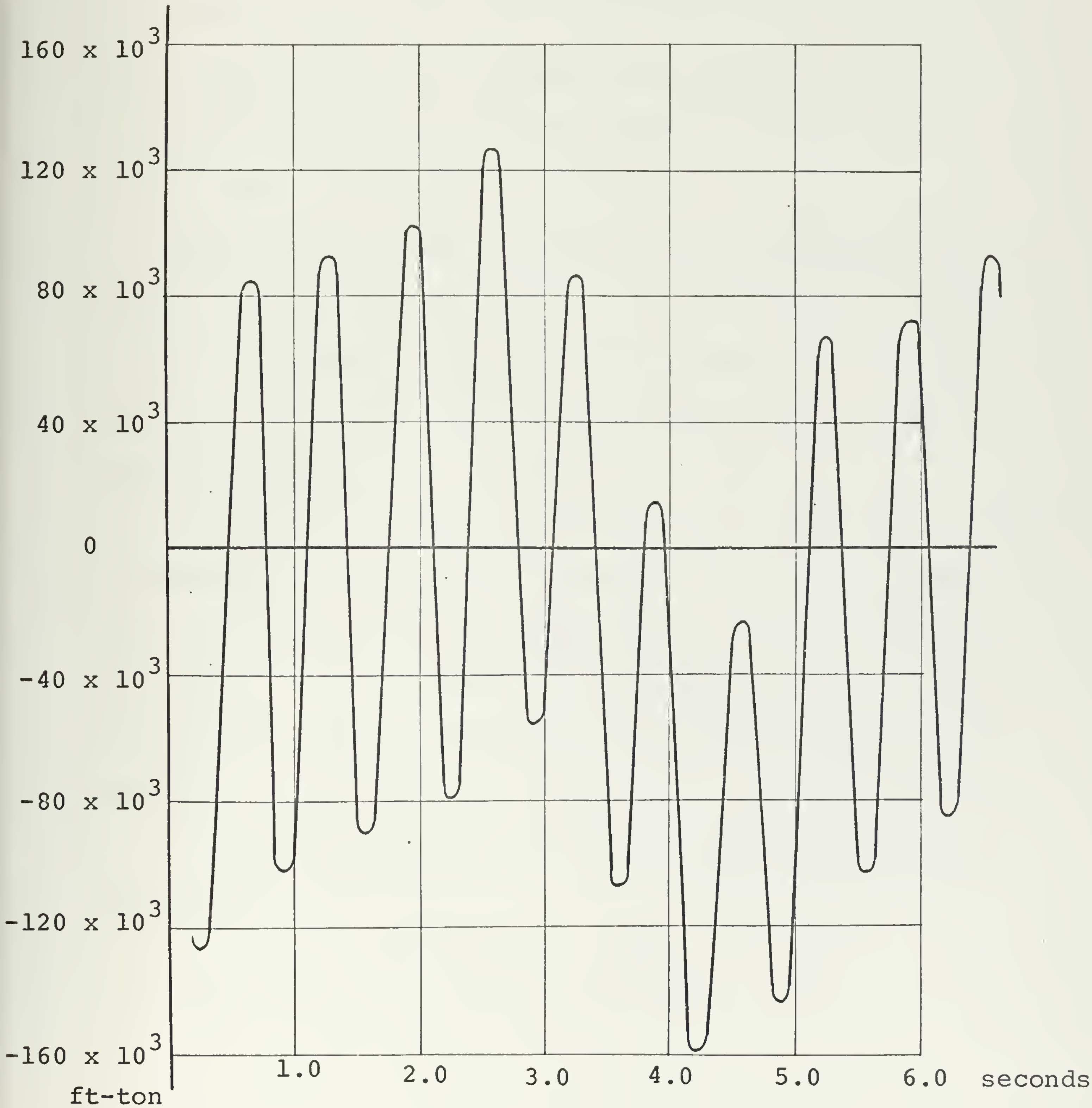

Figure 11 Midship bending moment for regular waves with w = 0.8 rad/sec

c) For w=0.6 rad/sec the variation is between approximately -500×10^3 ft-ton and 450×10^3 ft-ton.

d) Finally, for w=0.8 rad/sec the minimum value is -160×10^3 ft-ton and the maximum is 130×10^3 ft-ton.

These figures show a strong correlation between bending moment amplitude and frequency. The amplitude is a minimum for w=0.2 rad/sec, it is a maximum for w=0.6 rad/sec and it is of about the same order of magnitude for the remaining two frequencies.

Towing tank tests also show this kind of correlation for the MARINER [8, 33]. The most severe slamming was found to occur for values of wave length (WL) over ship length (L) of 1.0 to 1.25, and almost all the waves of WL/L = 0.75 to 2.25 and of heights greater than 20 ft were found to cause slamming.

In terms of WL/L we have for our four frequencies the following:

w (rad/sec)	WL/L
0.2	9.7190
0.4	3.4297
0.6	1.0799
0.8	0.6074

It is interesting to note that our results agree with the towing tank experiments reported above at least in two points:

a) The response is maximum for WL/L = 1.0799 (w = 0.6 rad/sec) which lies in the range of 1.0 to 1.25.

b) The response is minimum for WL/L = 9.7190 (w = 0.2 rad/sec) which lies outside the range 0.75 to 2.25.

Concerning the relative contributions of the different modes to the response we can conclude the following (as an illustration the three modes are shown for the particular case of w = 0.4 rad/sec in Figure 9):

a) For w = 0.2 rad/sec the second and third mode amplitudes can reach about 20% of the first mode amplitude.

b) For the remaining studied frequencies the importance of modes 2 and 3 decreases but their amplitudes still reach a maximum of about 10% of mode one's amplitude.

c) Neglecting the influence of modes 2 and 3 is premature, since their contribution to the overall response can still be significant.

Comparing our results with the experimental ship bending moments at midships for a 368 ft long destroyer at a speed of

17 knots [4], we can conclude they are of the same order of magnitude. In fact [4] gives a time history similar in shape to what we obtained, and the maximum value it reaches is approximately 50 x 10^3 ft-tons. We should note that this experimental response refers to a real situation of irregular seas (no Sea State given), where the ship's rigid body motions were taken from actual records.

Kaplan [7] obtains through simulation a time varying bending moment for the MARINER in irregular seas that seems to decay exponentially with time rather than having a certain periodicity. The maximum amplitude it reaches is about 35 x 10^3 ft-tons (for a speed of 12 knots and Sea State 7), which seems to be quite low compared with our values.

It is also possible to compare our results with the standard bending moment calculation. For the MARINER [35] gives these for a wave height of $1.1\sqrt{L}$, which is close to the figure of 15 ft amplitude used here. The most critical condition is for hogging where the value of the midship bending moment for the full-load condition is 388,300 ft-ton. We see then that the maximum midship bending moment due to slamming in regular waves may be larger than the maximum standard bending moment by as much as about 30% or more, for a particular wave frequency (in our case for w = 0.6 rad/sec).

We may conclude that the numerical results of application to the MARINER as described in this section are within an

acceptable order of magnitude. However, a more extensive comparison with experimental data should be carried out if more accurate conclusions were to be taken.

VI. CONCLUDING REMARKS

The numerical application to a MARINER of the midship slamming response method outlined in Sections I to IV showed, as already discussed, acceptable results within the ranges reported in the literature. It is necessary, however, to examine again the procedure and study the possibilities of expanding and refining it. An important point is weighting the validity of the several implied assumptions which include mainly:

a) the definition of the loading function P(x,t) as given in (6) with the Smith effect neglected;

b) the neglect of localized hydrodynamic effects, as air cushioning and spray;

c) the consideration of only vertical plane motions in head seas and the adoption of the Korvin-Kroukovsky equations;

d) the use of the beam theory normal mode of approach, with neglect of such problems as local effects, coupling of vibration modes and higher modes of vibration.

A desirable extension of the analysis should provide for the solution of the irregular seas condition as already stated, since this is the one likely to give more useful and realistic results.

The present work with all its imperfections arrived at a formulation and procedure that have the virtue of not being difficult to apply to specific cases, and this must be considered when estimating the weight of the above assumptions.

The computational sequence is in general easy to carry except for the preparation of data (m and A as a function of time for each station) for the so-called computer program II. Anybody with a fairly good programming experience may easily devise a quicker way of doing it. In fact programs II and III are extremely simple and it is possible to combine them with the ADMASS program avoiding then the lengthy intermediate tabulation and graphical representation of m as done in Figure 6. Also the tabulation of A (sectional area) may be avoided by some convenient description of the hull form, as done for example in subroutine OFFSET of [31]. The reason this was not carried out here is that our main objective was the discussion of the method itself and its formulation, rather than the preparation of a computational algorithm.

To conclude, we sincerely hope that in the near future the study of this subject is further developed and improved so that eventually the response of ships to any type of slamming may be readily obtained. If so, the benefits to be derived from this analysis would be enormous, with ship's structural design certainly closer to an optimum and safety and effectiveness undoubtedly increased.

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